$$\Delta V=2V\left|\left[2-\left(1-\frac{\tau\Delta\lambda}{360\,\Delta T}\right)^{2/3}\right]^{1/2}-1\right| \tag{A2}$$

$$\Delta V=2V\left|\left[2-\left(\frac{\tau_i}{\tau}\right)^{-2/3}\right]^{1/2}-1\right| \tag{A3}$$

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Analysis of Thick Cylinders with Internal Semicircular Grooves Subjected to Internal Pressure

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Introduction

SOLID propellant rocket motor grains are generally thick cylinders with internal axisymmetric grooves. Certain analysis and design modifications to reduce the effects of such grooves were investigated in Refs. 1 and 2. Essentially, the propellant grain is a viscoelastic material and a rigorous viscoelastic analysis is to be performed to assess the performance of the grain. Since the numerical procedure involved in the viscoelastic analysis is a series of elastic analyses in the time domain, a very accurate and reliable elastic analysis tool is necessary to get a satisfactory viscoelastic solution. And for configuration involving internal grooves, the continuum analysis becomes tedious and finite element methods can be advantageously used to get accurate stress pictures around the grooves. For this purpose, a six-degree-of-freedom triangular solid ring element is available in the literature,[3] but it is the authors' experience that the use of this element does not give satisfactory stress picture around the grooves even with a very fine mesh division. Then the alternative is to use the complex isoparametric elements[3] or high-precision triangular solid axisymmetric ring element,[4] to get a realistic solution around the grooves.

The main aim of the present Note is to get the elastic stress distribution using the high-precision triangular ring element[4] along the semi-circular internal grooves of a thick cylinder subjected to internal pressure, which is a typical sample of a rocket grain, for various groove geometries. As the emphasis given in the Note is the study of the variation of stress concentration along the groove, a typical configuration of a thick cylinder with internal grooves is considered and groove geometries alone are varied.

High Precision Finite Element

A typical ring element with triangular cross section is considered for the analysis of axisymmetric solids. The element has three nodes and six degrees of freedom per node, namely, u, u_x, u_y, v, v_x, v_y, where subscript denotes partial differentiation. The integration involved in the derivation of element matrices are evaluated numerically using the algorithm given in Ref. 5. The detailed derivation of element

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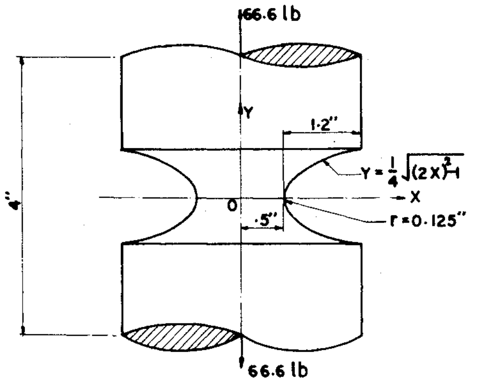

Fig. 1 Circular shaft under tension with deep hyperbolic notch.

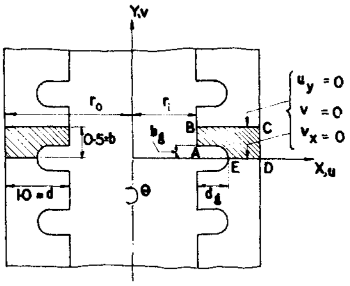

Fig. 2 Long, thick cylinder with equally spaced semi-circular grooves.

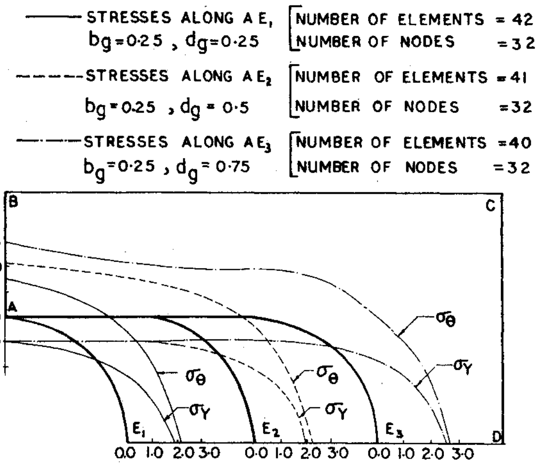

Fig. 3 Stress distribution along AE_1, AE_2 and AE_3.

stiffness matrix and the consistent load vectors of the high-precision element are presented in Ref. 4 and hence these details are not repeated here. A significant advantage of the present element over the element given in Ref. 3 is that all the strain components are included in the nodal degrees-of freedom and hence the evaluation of nodal strains and stresses is extremely simple.

To demonstrate the extreme accuracy and reliability of the present high-precision element, it is first applied to the standard Lame's thick cylinder problem and the results are presented in Table 1. It is evident from this table that the results obtained by the present element are extremely accurate even with a very coarse four element idealization.

Secondly, the problem of a shaft in tension with a deep hyperbolic notch (see Fig. 1) is considered to establish the efficiency of the present problem in predicting the stress concentration. The tangential stress at the base of the notch is calculated for three different sets of idealizations: a) 31 elements and 25 nodes; b) 31 elements and 25 nodes taking finer mesh around the notch; and c) 74 elements and 49 nodes. The stress concentration factor at the base of the notch for these three cases are found to be 2.771, 2.5887, and 2.3446,

Table 1 Thick cylinder subjected to unit internal pressure—displacements and stresses[a]

Radius, in.	Displacement, $u \times 10^{-7}$ in. Present element[b]	Displacement, $u \times 10^{-7}$ in. Exact[7]	Stresses, lb/in.^{2} σ_x Present element	σ_x Exact	σ_x Ref. (8)[c]	σ_θ Present element	σ_θ Exact	σ_θ Ref. (7)	σ_y Present element	σ_y Exact
1.0	1.9066	1.9066	−0.9797	−1.0	−0.840	1.6755	1.6667	1.734	0.2087	0.2000
1.5	1.4157	1.4156	−0.2582	−0.2593	−0.262	0.9264	0.9259	0.925	0.2004	0.2000
2.0	1.2133	1.2133	−0.0027	0.0000	−0.028	0.6654	0.6667	0.657	0.1988	0.2000

[a] Geometry: external radius/internal radius = 2.0; $E = 10^7$ lb/in.^{2}; $\nu = 0.3$; internal pressure = 1 lb/in.^{2}; u = radial displacement; σ_x = radial stress; σ_y = axial stress; and σ_θ = tangential stress. [b] Matrix order 12. [c] Matrix order 63 (using the six-degree-of-freedom finite element of Ref. 3).

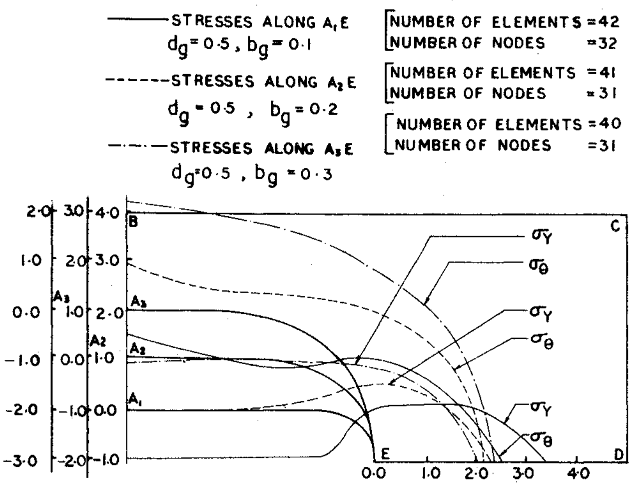

Fig. 4 Stress distribution along A_1E, A_2E and A_3E.

respectively. The theoretical value of Neuber[6] for an infinite shaft is 2.175. The agreement between these solutions is satisfactory.

Thick Cylinders with Internal Semicircular Grooves

Figure 2 shows a long thick cylinder with equally spaced semi-circular grooves, subjected to internal pressure of unity. The ratio of the external radius to the internal radius (r_o/r_i) of the cylinder is taken as two. The spacing between the center lines of the grooves $2b$ is taken as 1. $2b_g$ is the depth of the groove and d_g is the length of the groove in the radial direction. Since the cylinder is assumed long with equally spaced grooves, only the portion ABCDE is considered for the analysis applying proper boundary conditions on the edges BC and DE as shown in Fig. 2.

Two parametric studies are made in this Note: 1) keeping b_g to be 0.25, d_g is varied from 0.25, 0.5 to 0.75; and 2) keeping d_g to be 0.5, b_g is varied from 0.1, 0.2 to 0.3. The stresses σ_y and σ_θ are obtained along the edge AE for the preceding cases and are presented in Figs. 3 and 4, respectively. To obtain accurate stresses along the edge AE, a fine finite element mesh is taken around the groove. It can be concluded from Fig. 3 that as d_g is increased σ_y and σ_θ increases marginally at the point E where as at the point A there is a significant increase of σ_θ. From Fig. 4, it can be noted that as b_g increases, at point E there is not much appreciable change in the value of σ_θ, but σ_y decreases significantly. σ_y at point A should be -1 because of the applied unit internal pressure and is correctly predicted by the present finite element analysis.

Conclusions

The high-precision axisymmetric solid ring element[4] is used in analyzing the stresses around the internal semi-circular grooves of a typical thick cylinder such as a kick motor of a 4 stage rocket subjected to internal pressure. The application of the present element in Lame's problem shows the extreme accuracy of the finite element used. Even though the analysis presented in this Note is for simple groove geometries, the present finite element can be confidently applied to the analysis of complicated internal grooves, which are actually encountered in the design of rocket motor grains.

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Assessment of Pressure Port Erosion Effects

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I. Introduction

EROSION around forebody pressure port holes on R/V's employing ablative heat shields can cause erroneous flight test pressure to be recorded in turbulent flow. The erroneous pressures can be misinterpreted during the post flight analysis causing invalid conclusions to be drawn regarding the R/V performance.

Forebody pressure data are required to meet the mission objectives/goals of most R&D R/V flight test programs by determining the inflight forebody drag component of total drag. Accordingly, it is important to assess the effects of

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Index categories: Entry Vehicles and Landers; LV/M Flight Testing; Supersonic and Hypersonic Flow.

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